OCR

Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Given that $y = 4x^2 \ln x$, find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = \mathrm{e}^2$. **[5]**

2 By first using appropriate identities, solve the equation

$$5\cos 2\theta \operatorname{cosec}\theta = 2$$

for $0° < \theta < 180°$. **[6]**

3 **(i)** Use Simpson's rule with four strips to find an approximation to

$$\int_0^2 \mathrm{e}^{\sqrt{x}}\,\mathrm{d}x\ ,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Deduce an approximation to $\int_0^2 (1 + 10\mathrm{e}^{\sqrt{x}})\mathrm{d}x$. **[2]**

4 The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 2x^3 + 4 \quad \text{and} \quad \mathrm{g}(x) = \sqrt[3]{x - 10}\ .$$

(i) Evaluate $\mathrm{f}^{-1}(-50)$. **[2]**

(ii) Show that $\mathrm{fg}(x) = 2x - 16$. **[2]**

(iii) Differentiate $\mathrm{gf}(x)$ with respect to x. **[3]**

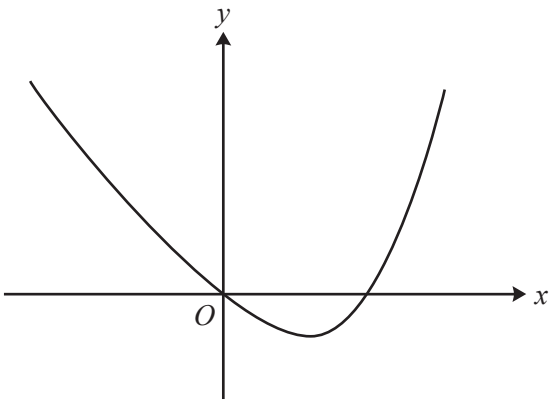
5 **(a)** The mass, M grams, of a substance at time t years is given by

$$M = 58\mathrm{e}^{-0.33t}\ .$$

Find the rate at which the mass is decreasing at the instant when $t = 4$. Give your answer correct to 2 significant figures. **[3]**

(b) The mass of a second substance is increasing exponentially. The initial mass is 42.0 grams and, 6 years later, the mass is 51.8 grams. Find the mass at a time 24 years after the initial value. **[4]**

6



The diagram shows the curve $y = x^4 - 8x$.

(i) By sketching a second curve on the copy of the diagram, show that the equation

$$x^4 + x^2 - 8x - 9 = 0$$

has two real roots. State the equation of the second curve. **[2]**

(ii) The larger root of the equation $x^4 + x^2 - 8x - 9 = 0$ is denoted by α.

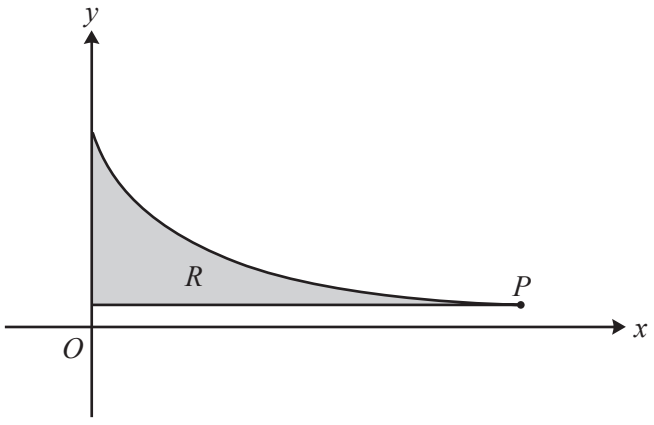
(a) Show by calculation that $2.1 < \alpha < 2.2$. **[2]**

(b) Use an iterative process based on the equation

$$x = \sqrt[4]{9 + 8x - x^2},$$

with a suitable starting value, to find α correct to 3 decimal places. Give the result of each step of the iterative process. **[4]**

7

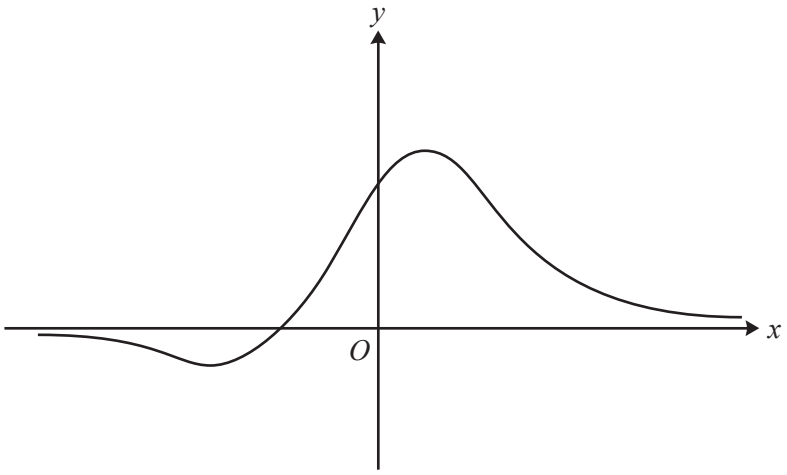


The diagram shows the curve $y = \sqrt{\dfrac{3}{4x+1}}$ for $0 \leqslant x \leqslant 20$. The point P on the curve has coordinates $\left(20, \frac{1}{9}\sqrt{3}\right)$. The shaded region R is enclosed by the curve and the lines $x = 0$ and $y = \frac{1}{9}\sqrt{3}$.

(i) Find the exact area of R. **[4]**

(ii) Find the exact volume of the solid obtained when R is rotated completely about the x-axis. **[6]**

8



The diagram shows the curve $y = \dfrac{2x+4}{x^2+5}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence find the coordinates of the two stationary points. **[6]**

(ii) The function g is defined for all real values of x by

$$\mathrm{g}(x) = \left|\frac{2x+4}{x^2+5}\right|.$$

(a) Sketch the curve $y = \mathrm{g}(x)$ and state the range of g. **[3]**

(b) It is given that the equation $\mathrm{g}(x) = k$, where k is a constant, has exactly two distinct real roots. Write down the set of possible values of k. **[2]**

9 **(i)** Express $5\cos(\theta - 60°) + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[4]**

(ii) Hence

(a) give details of the transformations needed to transform the curve $y = 5\cos(\theta - 60°) + 3\cos\theta$ to the curve $y = \sin\theta$, **[3]**

(b) find the smallest positive value of β satisfying the equation

$$5\cos\left(\tfrac{1}{3}\beta - 40°\right) + 3\cos\left(\tfrac{1}{3}\beta + 20°\right) = 3. \quad \textbf{[5]}$$

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			Attempt use of product rule to find first derivative	M1	producing form … ± … where one term involves $\ln x$ and the other does not	
			Obtain	A1	or unsimplified equiv	
			Attempt use of correct product rule to find second derivative	M1	with one term involving $\ln x$	
			Obtain $8\ln x + 12$	A1	or unsimplified equiv	
			Obtain 28	A1		
				[5]		
2			State or imply $\text{cosec}\,\theta = 1 \div \sin\theta$	B1	allow $\text{cosec} = 1 \div \sin$	
			Attempt to express equation in terms of $\sin\theta$ only	M1	using identity of form $\pm 1 \pm 2\sin^2\theta$ for $\cos 2\theta$	
			Obtain $10\sin^2\theta + 2\sin\theta - 5 = 0$	A1	or unsimplified equiv involving $\sin\theta$ only but with no $\sin\theta$ remaining in denominator	
			Attempt use of formula to find $\sin\theta$ from 3-term quadratic equation involving $\sin\theta$ (using formula or completing square even if their equation can be solved by factorisation)	M1	use implied by at least one correct value of $\sin\theta$ or θ; if correct quadratic formula quoted, condone one sign error for M1; if formula not first quoted, any error leads to M0	if completion of square used to solve equation, this must be correct for M1 to be earned
			Obtain 37.9°	A1	or greater accuracy 37.8896…	
			Obtain 142°	A1	or greater accuracy 142.1103…; and no others between 0 and 180; ignore any answers, right or wrong, outside 0 - 180	no working and answers only (max 2/6): 37.9 (or greater accuracy) B1 142 (or greater accuracy) and no others … B1
				[6]		

Question			Answer	Marks	Guidance	
3	(i)		Attempt calculation $k(y+4y+2y+...)$	M1	any constant k; using y values with coefficients 1, 2, 4 each occurring at least once; brackets may be implied by subsequent calculation	allow M1 for attempt using y values based on wrong x values such as 0, 1, 2, 3, 4; attempt based on $k(y_0+y_4)+4y_1+2y_2+4y_3$ is M0 unless subsequent calculation shows missing brackets are 'present'
			Obtain $k(e^0+4e^{\sqrt{0.5}}+2e+4e^{\sqrt{1.5}}+e^{\sqrt{2}})$	A1	or equiv perhaps involving decimal values 1, 2.02811…, 2.71828…, 3.40329…, 4.11325…	
			Use $k=\frac{1}{3}\times\frac{1}{2}$	A1		
			Obtain 5.38	A1	allow 5.379 but not, in final answer, greater 'accuracy'; answer $5.38+c$ is final A0	answer only: 0/4
				[4]		
3	(ii)		Attempt calculation of form $10\times$(answer to part i)$+k$	M1	implied by correct answer only or by answer following correctly from their incorrect part **(i)** ; any non-zero constant k	allow attempt involving second use of Simpson's rule: M1 for complete correct expression, A1 for answer
			Obtain 55.8 or greater accuracy based on their part **(i)** – more than 3 s.f. acceptable	A1ft	following their answer to part **(i)** but A0 for $55.8+c$	answer only 54.8 with no working earns M1A0 (as does 10(their ans) + 1); otherwise incorrect answer with no working earns 0/2
				[2]		
4	(i)		Either: State $2x^3+4=-50$	B1		
			State -3 and no other	B1		
			Or: Obtain $\sqrt[3]{\frac{1}{2}(x-4)}$ for inverse of f	B1	or equiv; using any letter	
			State -3 and no other	B1		
				[2]		
4	(ii)		Show composition of functions the right way round	M1		
			Obtain $2x-16$	A1	AG; necessary detail needed	first step $2(x-10)+4$ acceptable but then two more steps needed
				[2]		

Question			Answer	Marks	Guidance	
4	(iii)		Obtain $\sqrt[3]{2x^3-6}$ or $(2x^3-6)^{\frac{1}{3}}$ for $\mathrm{gf}(x)$	B1	or unsimplified equiv	
			Apply chain rule to function which is cube root of a non-linear expression	M1	condone incorrect constant; otherwise use of chain rule for their function must be correct	may use $u=2x^3-6$; M1 earned for expression involving u
			Obtain $2x^2(2x^3-6)^{-\frac{2}{3}}$	A1	or similarly simplified equiv; do not accept final answer with $\frac{6}{3}$ unsimplified	… in terms of x
				[3]		
5	(a)		Differentiate to produce $k\mathrm{e}^{-0.33t}$	M1	where constant k is different from 58	method must involve differentiation
			Obtain $-19.14\mathrm{e}^{-0.33t}$ or $19.14\mathrm{e}^{-0.33t}$	A1	or unsimplified equiv	
			Obtain -5.1 or 5.1	A1	whatever they claim value represents; accept 5.11 but not greater accuracy	
				[3]		
5	(b)		Either:			
			State or imply formula $42\mathrm{e}^{kt}$ or $42a^t$	B1	$42\mathrm{e}^{-kt}$, $42\mathrm{e}^{-kx}$, etc. also acceptable	
			Attempt to find k from $42\mathrm{e}^{6k}=51.8$ or a from $42a^6=51.8$	M1	using sound process involving logarithms at least as far as $6k=\ldots$ or $a=\ldots$	
			Obtain $k=0.035$ or $a=1.0356$	A1	or greater accuracy 0.03495… or exact equiv $\frac{1}{6}\ln\frac{37}{30}$	
			Substitute 24 to obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.	
			Or:			
			Use ratio $\frac{51.8}{42}$ in calculation	B1		
			Attempt calculation of form $42\times r^n$	M1		
			Obtain $42\times(\frac{51.8}{42})^4$ or $51.8\times(\frac{51.8}{42})^3$	A1		
			Obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.	
				[4]		

Question			Answer	Marks	Guidance	
6	(i)		Draw inverted parabola roughly symmetrical about the *y*-axis and with maximum point more or less on *y*-axis State $y = 9 - x^2$ and indicate two intersections by marks on diagram or written reference to two intersections	M1 A1 **[2]**	drawing enough of the parabola that two intersections occur, ignoring their locations at this stage now needs second curve drawn so that right-hand intersection occurs in first quadrant	
6	(ii)	(a)	Calculate values of quartic expression for 2.1 and 2.2 Obtain –1.9... and 1.6… and draw attention to sign change or clear equiv	M1 A1 **[2]**	if no explicit working seen, M1 is implied by at least one correct value; but if no explicit working seen and both values wrong, award M0	
6	(ii)	(b)	Obtain correct first iterate Carry out process to produce at least three iterates in all Obtain at least two more correct iterates Obtain 2.156	B1 M1 A1 A1 **[4]**	starting anywhere between –1 and 9 and showing at least 3 d.p. implied by plausible sequence of values; allow recovery after error showing at least 3 decimal places final answer needed to exactly 3 d.p.; not given for 2.156 as final iterate in sequence, i.e. needs indication (perhaps just underlining) that value of α found	$2.1 \to 2.15056 \to 2.15531 \to 2.15575 \to 2.15579$ $2.15 \to 2.15526 \to 2.15574 \to 2.15579$ $2.2 \to 2.15980 \to 2.15616 \to 2.15583 \to 2.15580$ answer only: 0/4

Question			Answer	Marks	Guidance	
7	(i)		Integrate to obtain $k(4x+1)^{\frac{1}{2}}$ or $ku^{\frac{1}{2}}$	*M1	any constant k	
			Obtain correct $\frac{1}{2}\sqrt{3}(4x+1)^{\frac{1}{2}}$ or $\frac{1}{2}\sqrt{3}u^{\frac{1}{2}}$	A1	or exact equiv	
			Apply limits 0 and 20 and attempt subtraction of area of rectangle (or limits 1 and 81 if u involved)	M1	dep *M; or equiv such as including term $-\frac{1}{9}\sqrt{3}$ in the integration or finding $\int \frac{1}{9}\sqrt{3}\,dx$ separately; allow M1 if decimal values used here	Alternative: (region between curve and y-axis) Obtain equation $x=\frac{3}{4}y^{-2}-\frac{1}{4}$ B1 Integrate to obtain form $k_1y^{-1}+k_2y$ *M1 Apply limits $\frac{1}{9}\sqrt{3}$ and $\sqrt{3}$ the right way round M1 d*M
			Obtain $4\sqrt{3}-\frac{20}{9}\sqrt{3}$ and hence $\frac{16}{9}\sqrt{3}$	A1	answer must be exact and a single term; $\frac{16}{9}\sqrt{3}+c$ as answer is final A0	Obtain $\frac{6}{\sqrt{3}}-\frac{8}{36}\sqrt{3}$ or better A1
				[4]		
	(ii)		State volume is $\pi\int\frac{3}{4x+1}\,dx$	B1	no need for limits here; condone absence of dx; condone absence of π here if it appears later in solution	allow B1 for $\int\pi y^2$ and $y^2=\frac{3}{4x+1}$ stated
			Obtain integral of form $k\ln(4x+1)$	M1	any constant k with or without π	if brackets missing, and subsequent calculation does not show their 'presence', marks are max B1M1A0A0M1A0
			Obtain $\frac{3}{4}\pi\ln(4x+1)$ or $\frac{3}{4}\ln(4x+1)$	A1		
			Apply limits to obtain $\frac{3}{4}\pi\ln 81$ or $\frac{3}{4}\ln 81$	A1	or exact equiv perhaps with ln1 present	
			Attempt to subtract volume of cylinder, using correct radius and 'height'	M1	with exact volume of cylinder attempted	do not treat rotation around y-axis as mis-read: this is 0/6
			Obtain $3\pi\ln 3-\frac{20}{27}\pi$ or $\pi(\frac{3}{4}\ln 81-\frac{20}{27})$	A1	or exact equiv involving two terms	
				[6]		

Question			Answer	Marks	Guidance	
8	(i)		Attempt use of quotient rule or equiv	M1	condone one slip only but must be subtraction in numerator; condone absence of necessary brackets; or equiv	
			Obtain $\frac{2(x^2+5)-2x(2x+4)}{(x^2+5)^2}$	A1	or correct equiv; now with brackets as necessary	correct numerator but error in denominator: max M1A0A1M1A1A1; numerator wrong way round: max M0A0A0M1A1A1
			Obtain $-2x^2-8x+10=0$	A1	or equiv involving three terms	
			Attempt solution of three-term quadratic equation based on numerator of derivative (even if their equation has no real roots)	M1	implied by no working but 2 correct values obtained	M1 for factorisation awarded if attempt is such that x^2 term and one other term correct upon expansion; if formula used, M1 awarded as per Qn 2
			Obtain -5 and 1	A1		
			Obtain $(-5, -\frac{1}{5})$ and (1, 1)	A1	Allow $-\frac{6}{30}$	
				[6]		
	(ii)	(a)	Sketch (more or less) correct curve	B1	showing negative part reflected in x-axis and positive part unchanged; ignore intercept values on axes, right or wrong	
			State values between 0 and their y-value of maximum point lying in first quadrant	M1	accept $\leq$ or $<$ signs here	
			State correct $0 \leq y \leq 1$	A1ft	following their y-value of maximum point in first quadrant; now with $\leq$ signs; or equiv perhaps involving g or g(x)	for "$y \geq 0$ and $y \leq 1$", award M1A1; for separate statements $y \geq 0$, $y \leq 1$, award M1A0
				[3]		
	(ii)	(b)	Indicate, in some way, values between y-coordinates of maximum point and reflected minimum point (provided their y-coordinate of minimum point is negative)	M1	allow $\leq$ sign(s) here; could be clear indication on graph	for "$k > \frac{1}{5}$ and $k < 1$", award M1A1; for separate statements, award M1A0
			State $\frac{1}{5} < k < 1$	A1	or correct equiv; not $\leq$ now; correct answer only earns M1A1	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)		Simplify to obtain $\frac{11}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$	B1	or equiv with two terms perhaps with $\sin 60$ retained	accept decimal values
			Attempt correct process to find R	M1	for expression of form $a\cos\theta + b\sin\theta$	obtained after initial simplification
			Attempt correct process to find α	M1	for expression of form $a\cos\theta + b\sin\theta$; condone $\sin\alpha = \frac{11}{2}$, $\cos\alpha = \frac{5}{2}\sqrt{3}$	obtained after initial simplification
			Obtain $7\sin(\theta + 51.8)$	A1	or greater accuracy 51.786…	
				[4]		
	(ii)	(a)	State stretch and translation in either order	M1	or equiv but using correct terminology, not move, squash, …	SC: if M0 but one transformation completely correct, award B1 for 1/3
			State stretch parallel to y-axis with factor $\frac{1}{7}$	A1ft	following their R and clearly indicating correct direction	
			State translation parallel to θ-axis or x-axis by 51.8 in positive direction or state translation by vector $\begin{pmatrix} 51.8 \\ 0 \end{pmatrix}$	A1ft	following their α and clearly indicating correct direction; or equiv such as 308.2 parallel to x-axis in negative direction	
				[3]		
		(b)	State left-hand side (their R) $\sin(\frac{1}{3}\beta + \gamma)$ where $\gamma \neq \pm$(their α), $\gamma \neq \pm 40$, $\gamma \neq \pm 20$	M1	or equiv such as stating $\theta = \frac{1}{3}\beta + 20$	
			Obtain (their R) $\sin(\frac{1}{3}\beta +$ their $\alpha + 20) = 3$	A1ft	(and, in this case, allowing A1ft provided value of $\frac{1}{3}\beta$ attempted later)	
			Attempt correct process to find any value of $\frac{1}{3}\beta$	M1	for equation of form $\sin(\frac{1}{3}\beta + \gamma) = k$ where $\lvert k\rvert < 1$, $k \neq 0$	
			Attempt complete process to find positive value of β	M1	including choosing second quadrant value of their $\sin^{-1}\frac{3}{7}$	
			Obtain 248 or 249 or 248.5	A1	or greater accuracy 248.508…	
				[5]		